ECE 440
Lecture 9: Carrier Concentrations and the Temperature Dependence

Class Outline:

• Intrinsic Carrier Concentrations
• Extrinsic Carrier Concentrations
• Thermal Effects Revisited
Key Questions

• What is the charge neutrality relationship?

• How are the carrier concentrations determined for extrinsic material?

• What role does temperature play in carrier concentrations in extrinsic material?

• What is compensated material?
Intrinsic Carrier Concentrations

We recall that by using the **density of states** and the **Fermi function**...

We know how many carriers we have in our semiconductor for **ANY** energy or temperature.

**For electrons:**

\[
 n = \int_{E_c}^{E_{top}} g_c(E)f(E)dE
\]

**For holes:**

\[
 p = \int_{E_{bottom}}^{E_v} g_v(E)[1-f(E)]dE
\]

We can simplify these integrals to find closed form relations...

- Effective conduction band density of states.

\[
 N_c = 2\left[ \frac{m_n^*k_bT}{2\pi\hbar^2} \right]^{\frac{3}{2}}
\]

- Effective valence band density of states.

\[
 N_v = 2\left[ \frac{m_p^*k_bT}{2\pi\hbar^2} \right]^{\frac{3}{2}}
\]

- Non-degenerate Semiconductor

\[
 N_{c,v} = \left( 2.51 \times 10^{19} \frac{1}{cm^3} \right) \left( \frac{m_{n,p}^*}{m_0} \right)^{\frac{3}{2}}
\]

- Degenerate Semiconductor

\[
 n = N_c e^{\frac{(E_f-E_c)}{k_bT}}
\]

\[
 p = N_v e^{\frac{(E_v-E_f)}{k_bT}}
\]
Intrinsic Carrier Concentrations

Is this the best and easiest??

\[ F_{1/2}(\eta_c) = \int_0^\infty \frac{\eta^2}{1 + e^{\eta - \eta_c}} \, d\eta \]

In two regions of temperature and doping concentration the Fermi-Dirac integral is easily performed:

Low temperature, highly degenerate:

\[ F_{1/2}(\eta_c) = \frac{2}{3} \eta_c^3 = \frac{2}{3} \left( \frac{E_f - E_c}{k_b T} \right)^{2/3} \]

High temperature, non-degenerate:

\[ F_{1/2}(\eta_c) = e^{\frac{(E_f - E_c)}{k_b T}} \]

Joyce-Dixon Approximation:

\[ E_f = E_c + k_b T \left[ \ln \frac{n}{N_c} + \frac{1}{\sqrt{8}} \frac{n}{N_c} \right] = E_v - k_b T \left[ \ln \frac{p}{N_v} + \frac{1}{\sqrt{8}} \frac{p}{N_v} \right] \]

But we can use curve fitting (no physics) to cover all of the ranges:
Intrinsic Carrier Concentrations

Recall that we can also find the dependence on temperature...

- We can find the carrier concentrations relative to $E_i$, the Fermi level for an intrinsic semiconductor.
- These relations result in the dependence of carrier concentration on the band gap.

For intrinsic semiconductors, we know the following: $n = p = n_i$ and $E_i = E_f$

Then the relations for $n$ and $p$ become:

$$n_i = N_c e^{\frac{E_i - E_c}{k_bT}}$$

$$n_i = N_v e^{\frac{E_v - E_i}{k_bT}}$$

By combining equations, we arrive at two very important and useful results:

$$n = n_i e^{\frac{E_f - E_i}{k_bT}}$$

$$p = n_i e^{\frac{E_i - E_f}{k_bT}}$$

Solve for $N_c$ and $N_v$

$$N_c = n_i e^{\frac{E_c - E_i}{k_bT}}$$

$$N_v = n_i e^{\frac{E_i - E_v}{k_bT}}$$

Solve for $N_i$

$$n_i = \sqrt{N_c N_v e^{\frac{-E_g}{2k_bT}}}$$
Extrinsic Carrier Concentrations

In most situations, we are dealing with extrinsic semiconductors...

**Donor** Doped Semiconductor Band Diagram:

\[ E_d \]
\[ \Delta x \]
\[ E_v \]

**Acceptor** Doped Semiconductor Band Diagram:

\[ E_a \]
\[ E_v \]
Extrinsic Carrier Concentrations

Let’s consider a small separate parts of a uniformly doped semiconductor...

What happens if we look at small separate parts of a non-uniformly doped semiconductor...
Extrinsic Carrier Concentrations

The preceding analysis leads to the **charge neutrality relationship**...

The total charge inside any region of semiconductor must be zero:
- Otherwise electric fields are present which break equilibrium.
- Charge motion and currents would be present.

\[
\frac{\text{charge} \, e}{cm^3} = qp - qn + qN_D^+ - qN_A^- = 0
\]

If there is sufficient thermal energy in the system, then we can simplify things further:

\[
N_D^+ = N_D
\]
\[
N_A^- = N_A
\]
\[
p - n - N_D - N_A = 0
\]
Extrinsic Carrier Concentrations

Let’s use the **charge neutrality relation**...

Here, we make two critical assumptions in our analysis of **extrinsic carrier concentrations**...

- We are dealing with non-degenerate semiconductors.
- All of the dopants are ionized.

Start with the np product expression:

\[ p = \frac{n_i^2}{n} \]

\[ \frac{p - n - N_D - N_A}{n} = 0 \]

\[ \frac{n_i^2}{n} - n + N_D - N_A = 0 \]

\[ n^2 - n(N_D - N_A) - n_i^2 = 0 \]

Solve quadratic equation

\[ n = \frac{N_D - N_A}{2} + \left[ \left( \frac{N_D - N_A}{2} \right)^2 + n_i^2 \right]^{\frac{1}{2}} \]

\[ p = \frac{N_A - N_D}{2} + \left[ \left( \frac{N_A - N_D}{2} \right)^2 + n_i^2 \right]^{\frac{1}{2}} \]
Extrinsic Carrier Concentrations

The previous equations are generic, we can simplify in 4 cases...

Case # 1: Intrinsic Semiconductor \( N_A = N_D = 0 \)

\[ n = p = n_i \]

Case # 2: Doped Semiconductor with \( N_D - N_A \sim N_D \gg n_i \) or \( N_A - N_D \sim N_A \gg n_i \)

\[ n \approx N_D \quad \text{or} \quad p \approx N_A \]

\[ p = \frac{n_i^2}{N_D} \quad \text{Assuming:} \]

• Non-degenerate
• Total ionization

\[ n = \frac{n_i^2}{N_A} \]

Case # 3: Doped Semiconductor with \( n_i \gg |N_A - N_D| \)

\[ n = p = n_i \]

All semiconductors become intrinsic at sufficiently high temperatures
Extrinsic Carrier Concentrations

What about the 4th case? Compensated Semiconductors

• Semiconductors can contain both acceptors and donors.
• Electrons seek the lowest possible energy state.
• At 300K most donors are ionized but some electrons in CB can seek lower energy states by ionizing acceptors.
• Donors have donated electrons to CB so they are *ionized* (+).
• Acceptors can accept electrons creating holes in VB, so they are *ionized* (-). But each acceptor ionized by an electron from the CB (i.e. from the donors) *cannot* create a hole in VB.

\[
p_0 + N_d^+ = n_0 + N_a^-
\]

Sum of positive charges (holes and ionized donors) must balance sum of negative charges (electrons and ionized acceptors):
Extrinsic Carrier Concentrations

Where is the intrinsic level, $E_i$, really located?

In Lecture 8, we saw that the intrinsic Fermi level was located at mid-gap. Does our current knowledge change anything?

In intrinsic material, we know that $n=p$. Now use the Boltzmann equations:

$$\frac{E_i - E_f}{k_B T} N_c e^{\frac{E_f - E_i}{k_B T}} = N_v e^{\frac{E_f - E_i}{k_B T}}$$

But,

$$\frac{N_v}{N_c} = \left( \frac{m_p^*}{m_n^*} \right)^{\frac{3}{2}}$$

So,

$$E_i = \frac{E_c + E_v}{2} + \frac{3}{4} k_B T \ln \left( \frac{m_p^*}{m_n^*} \right)$$

Only when the masses are equal or $T = 0 \text{ K}$ does $E_i$ lie at midgap!
Extrinsic Carrier Concentrations

So where does the Fermi level lie for a doped semiconductor??

Again, we make the assumption that the semiconductor is non-degenerate, maintained in equilibrium at temperatures where all of the dopants are ionized...

\[ E_f - E_i = \frac{E_i - E_f}{k_B T} \]

\[ n = n_i e^{\frac{E_f - E_i}{k_B T}} \]

\[ p = n_i e^{\frac{E_i - E_f}{k_B T}} \]

In a typical doped semiconductor, where \( n \sim N_D \) or \( p \sim N_A \)...

\[ E_f - E_i = k_B T \ln \left( \frac{N_D}{n_i} \right) \quad \text{for} \quad N_D \gg N_A, N_D \gg n_i \]

\[ E_i - E_f = k_B T \ln \left( \frac{N_A}{n_i} \right) \quad \text{for} \quad N_A \gg N_D, N_A \gg n_i \]
Extrinsic Carrier Concentrations

So where does the Fermi level lie for a doped semiconductor??

Fermi level positioning in Si at 300 K as a function of the doping concentration...
Thermal Effects Revisited

What is the role of temperature??

Temperature defines three distinct regions of conductivity...