ECE 340
Lecture 24 : Quantitative Current Flow in a P-N Junction

Class Outline:

• Quantitative PN Junction Current
Key Questions

• How do we calculate excess carriers under bias?
• How do we calculate the total current?
• What happens to the current far away from the junction?
Quantitative PN Junction Current

Taking a closer look at the forward and reverse bias carrier concentrations...

Forward Bias

Reverse Bias

Physics of Semiconductor Devices, S.M. Sze, Wiley-Interscience
Quantitative PN Junction Current

Take a closer look at the **forward bias** regime...

Forward bias increases the probability of diffusion across the junction exponentially.

\[ I = I_0(e^{qV/kT} - 1) \]

Total current is the diffusion current minus the absolute value of the generation current.

At \( V = 0 \), the generation and diffusion currents cancel.

End result is a rectifying type of behavior seen in MS contacts.

*I-V characteristic of a p-n junction*
To understand the specifics about how current flows, we need to make a few assumptions...

1. The diode is being operated under steady state conditions.
3. The diode is one dimensional.
4. Low level injection prevails in the quasi-neutral regions.
5. There are no processes other than:
   - Drift
   - Diffusion
   - Thermal recombination and generation

What general relationships will we need?

\[ I = AJ \quad (A = \text{cross-sectional area}) \]

\[ J = J_N(x) + J_P(x) \]

\[ J_N = q\mu_n n \xi + qD_N \frac{dn}{dx} \]

\[ J_P = q\mu_p p \xi - qD_P \frac{dp}{dx} \]

Constant current but concentrations vary with position
Let’s start examining the different areas of the pn diode…

Start in the **quasi-neutral regions**. We have the minority carrier diffusion equations…

\[ 0 = D_N \frac{d^2 \Delta n_p}{dx^2} - \frac{\Delta n_p}{\tau_n} \quad x \leq -x_p \]

\[ 0 = D_p \frac{d^2 \Delta p_n}{dx^2} - \frac{\Delta p_n}{\tau_p} \quad x \geq x_n \]

But we know the electric field in the quasi-neutral regions is very small and this has consequences…

\[ \frac{dn_0}{dx} = \frac{dp_0}{dx} = 0 \]

\[ J_N = qD_N \frac{d\Delta n_p}{dx} \quad x \leq -x_p \]

\[ J_p = -qD_p \frac{d\Delta p_n}{dx} \quad x \geq x_n \]
Quantitative PN Junction Current

However, to determine the total current we need to know \( J_n \) and \( J_p \) at one point...the **depletion region**.

We must solve the continuity equations since the electric field is non-zero...

\[
0 = \frac{1}{q} \frac{dJ_N}{dx} + \frac{\partial n}{\partial t} \bigg|_{\text{thermal}} \\
0 = -\frac{1}{q} \frac{dJ_p}{dx} + \frac{\partial p}{\partial t} \bigg|_{\text{thermal}}
\]

For now, assume that there is no thermal recombination or generation in the depletion region. Now simplify...

\[
\begin{align*}
\frac{dJ_N}{dx} &= 0 \\
\frac{dJ_p}{dx} &= 0
\end{align*}
\]

\[
J_N(-x_p \leq x \leq x_n) = J_N(-x_p) \\
J_p(-x_p \leq x \leq x_n) = J_p(x_n)
\]

\[
J = J_N(-x_p) + J_p(x_n)
\]

Only need solutions at the edges!
We also need the boundary conditions for the minority carrier concentrations at the contacts...

\[ \Delta n_p(x \to -\infty) = 0 \]
\[ \Delta p_n(x \to +\infty) = 0 \]

What about at the edge of the depletion region?

Many diffusion lengths

We can use the quasi-Fermi levels.

\[ np = n_i^2 e^{(F_N - F_P)/kT} \]

Monotonic variation allows simplification...

\[ F_N - F_P \leq E_{Fn} - E_{Fp} = qV_A \]

\[ np = n_i^2 e^{qV_A/kT} \quad \ldots -x_p \leq x \leq x_n \]
Quantitative PN Junction Current

We still don’t have the boundary conditions at the edges of the depletion region...

Evaluate at the edge of the p-region...

\[ n(-x_p) = \frac{n_i^2}{N_A} e^{qV_A/kT} \]

Now simplify:

\[ \Delta n_p(-x_p) = \frac{n_i^2}{N_A} \left( e^{qV_A/kT} - 1 \right) \]

Similarly on the n-side:

\[ n(x_n)p(x_n) = p(x_n)N_D = n_i^2 e^{qV_A/kT} \]

Simplify again:

\[ \Delta p_n(x_n) = \frac{n_i^2}{N_D} \left( e^{qV_A/kT} - 1 \right) \]

Again, this was done by assuming that:

\[ F_N - F_P = qV_A \]

or

\[ F_N = E_{Fn} \quad \text{and} \quad F_P = E_{Fp} \]

Constant Fermi levels in the depletion region.
Now let’s spell out the game plan for solving pn junction problems under bias...

1. Solve minority carrier diffusion equations using the boundary conditions at the edge of the depletion region and contacts to obtain the decay of the minority carrier concentrations.

2. Compute the minority carrier current densities in the quasi-neutral regions.

3. Find the solutions for the currents at the edges of the depletion region and add the current densities at each edge of the depletion region.

Let’s begin with the proper derivation
Let's start with holes on the quasi-neutral n-region...

To simplify the math, shift the origin to the n-edge: $x' \geq 0$

Now apply boundary conditions:

$$0 = D_p \frac{d^2 \Delta p_n}{dx'^2} - \frac{\Delta p_n}{\tau_p}$$

$$\Delta p_n(x' \to \infty) = 0$$

$$\Delta p_n(x' = 0) = \frac{n_i^2}{N_D} (e^{qV_A/kT} - 1)$$

From our previous discussions about the solution to the diffusion equation, we already know the general form of the solution...

$$\Delta p_n(x') = A_1 e^{-x'/L_p} + A_2 e^{x'/L_p}$$

$$L_p = \sqrt{D_p \tau_p}$$

We can simplify by getting rid of $A_2$.

$$J_p(x') = -qD_p \frac{d\Delta p_n}{dx'} = q \frac{D_p n_i^2}{L_p N_D} (e^{qV_A/kT} - 1) e^{-x'/L_p}$$
Now shift the origin again and deal with the p-side of the junction...

And we get the same forms of the equations for the minority carrier concentrations and the current density:

\[
\Delta n_p(x'') = \frac{n_i^2}{N_A} \left( e^{qV_A/kT} - 1 \right) e^{-x''/L_N}
\]

\[
J_N(x'') = -qD_N \frac{d\Delta n_p}{dx''} = q \frac{D_N}{L_N} \frac{n_i^2}{N_A} \left( e^{qV_A/kT} - 1 \right) e^{-x''/L_N}
\]

Now evaluate the current at the edges of the depletion region and sum the contributions:

\[
J_N(x = -x_p) = J_N(x'' = 0) = q \frac{D_N}{L_N} \frac{n_i^2}{N_A} \left( e^{qV_A/kT} - 1 \right)
\]

\[
J_P(x = x_n) = J_P(x' = 0) = q \frac{D_P}{L_P} \frac{n_i^2}{N_D} \left( e^{qV_A/kT} - 1 \right)
\]

Finally,

\[
I = AJ = qA \left( \frac{D_N}{L_N} \frac{n_i^2}{N_A} + \frac{D_P}{L_P} \frac{n_i^2}{N_D} \right) \left( e^{qV_A/kT} - 1 \right)
\]

\[
I_0 = qA \left( \frac{D_N}{L_N} \frac{n_i^2}{N_A} + \frac{D_P}{L_P} \frac{n_i^2}{N_D} \right)
\]
Quantitative PN Junction Current

So what happens to the carriers...

• We expect minority carrier concentrations on each side to vary with the applied bias.

• Due to the variations in the diffusion of carriers across the junction

\[
p_p = \frac{e^{qV_0/kT}}{p_n}
\]

Equilibrium hole concentrations on each side of the barrier.

With bias this becomes:

\[
\frac{p(-x_{p0})}{p(x_{n0})} = e^{q(V_0-V)/kT}
\]

Altered barrier

• Valid for forward and reverse bias

• We assume low level injection - majority charge concentrations do not change.
With **low level injection**, we can begin to simplify...

• The carrier concentrations are still changing despite the fact that we are neglecting the changes in the majority concentrations.

• **Absolute increase** of the majority concentrations means that there is an increase of the minimum concentrations ($p_p$ and $n_n$) required to maintain charge neutrality.

• **Relative changes** in $p_p$ and $n_n$ vary only slightly compared to equilibrium values $p_0$ and $n_0$.

Take the ratio of the concentrations with and without bias...

\[
\frac{p(x_n0)}{p_n} = e^{\frac{qV}{nkT}} \quad \text{assuming} \quad p(-x_p0) = p_p
\]

What does this equation **tell us**?
This equation gives us insight into the **carrier concentration behavior under bias conditions**...

\[ \frac{p(x_{n0})}{p_n} = e^{qV_i/kT} \]

**Under forward bias:**
the equation suggests a greatly increased hole concentration at the edge of the n-side.

Conversely, the hole concentration under reverse bias is much smaller than the equilibrium value.

Exponential increase in hole concentration at \( x_{n0} \) with forward bias is an example of **minority Quantitative PN Junction Current**.
We can determine the **excess electrons and holes**...

Subtract the equilibrium concentrations...

\[
\frac{p_p}{p_n} = e^{qV_0/kT}
\]

From the concentrations under bias...

\[
\frac{p(x_{n0})}{p_n} = e^{qV/kT}
\]

\[
\Delta p_n = p(x_{n0}) - p_n = p_n(e^{qV/kT} - 1)
\]

\[
\Delta n_p = n(-x_{p0}) - n_p = n_p(e^{qV/kT} - 1)
\]

- Should produce a distribution of excess holes in the n material.
- As the holes diffuse, the recombine so the solution is identical to the **diffusion equation**.
So we can write down the solution to the diffusion equation on either side of the junction...

Excess electrons on p-side:

\[ \delta n(x_p) = \Delta n_p e^{-x_p/L_n} = n_p(e^{qV/kT} - 1)e^{-x_p/L_n} \]

Excess holes on n-side:

\[ \delta p(x_n) = \Delta p_n e^{-x_n/L_p} = p_n(e^{qV/kT} - 1)e^{-x_n/L_p} \]

Now we understand the hole diffusion current at any point...

\[ I_p(x_n) = -qAD_p \frac{d\delta p(x_n)}{dx_n} = qA \frac{D_p}{L_p} \Delta p_n e^{-x_n/L_p} = qA \frac{D_p}{L_p} \delta p(x_n) \]

Hole diffusion proportional to excess hole concentration. So what is the total current injected into the n-material?

\[ I_p(x_n = 0) = \frac{qAD_p}{L_p} \Delta p_n = \frac{qAD_p}{L_p} p_n(e^{qV/kT} - 1) \]

\[ I_n(x_p = 0) = -\frac{qAD_n}{L_n} \Delta n_p = -\frac{qAD_n}{L_n} n_p(e^{qV/kT} - 1) \]

Minus arises from current being directed opposite to \( x_p \).
Quantitative PN Junction Current

Take \( +x \) as the reference direction, what is the total current?

The total current must be the sum of the electron and hole contributions...

\[
I = I_p(x_n = 0) - I_n(x_p = 0) = \frac{qAD_p}{L_p} \Delta p_n + \frac{qAD_n}{L_n} \Delta n_p
\]

Which can be simplified to the Diode Equation...

\[
I = qA\left(\frac{D_p}{L_p} p_n + \frac{D_n}{L_n} n_p\right)\left(e^{qV/kT} - 1\right) = I_0\left(e^{qV/kT} - 1\right)
\]

In arriving at this equation:
- We have made no assumptions as to the sign of the bias voltage.
- Bias may be either forward or reverse
Quantitative PN Junction Current

Let’s check reverse bias ($V = -V_r$)... 

The total current then becomes:

$$I = qA \left( \frac{D_p}{L_p} p_n + \frac{D_n}{L_n} n_p \right) \left( e^{-\frac{qV_r}{kT}} - 1 \right)$$

If $V_r$ is larger than a few $kT/q$:

$$I = -qA \left( \frac{D_p}{L_p} p_n + \frac{D_n}{L_n} n_p \right) = -I_0$$

But what does the diode equation imply?

• The current is dominated by the injection of carriers from the more heavily doped side.

• We can, in most cases, simplify the diode equation to include only the contribution from the more heavily doped side.
But remember the **Fermi levels**...

We are out of equilibrium, so we need to use the quasi-Fermi levels to calculate the carrier concentrations...

\[ pn = n_i^2 e^{(F_n - F_p)/kT} = n_i^2 e^{(qV/kT)} \]

- Minority carrier concentration usually varies the most and the majority carrier quasi-Fermi level is close to the original Fermi level.
- Outside the space charge region the quasi-Fermi levels vary linearly and then merge with the bulk Fermi levels.
Is there another way to calculate the current?

• Assume the current supplies the excess carriers in the distributions.

• $I_p$ must supply enough holes per second to maintain the steady-state.

We can determine the total positive charge stored in the excess carrier distribution...

$$Q_p = qA \int_0^\infty \delta p(x_n) dx_n = qA \Delta p_n \int_0^\infty e^{-x_n/L_p} dx_n = qAL_p \Delta p_n$$

Charge that recombines must then be resupplied...

$$I_p(x_n = 0) = \frac{Q_p}{\tau_p} = qA \frac{L_p}{\tau_p} \Delta p_n = qA \frac{D_p}{L_p} \Delta p_n$$

• Solve for negative charge to get $\tau_n$.

Charge Control Approximation
There are two ways to calculate current...

\[ I_n(x_p=0) = qAD_n \frac{d\delta n}{dx_p} \bigg|_{x_p=0} \]

\[ I_p(x_n=0) = -qAD_p \frac{d\delta p}{dx_n} \bigg|_{x_n=0} \]

\[ I_n(x_p=0) = -qA \frac{D_n}{L_n} \Delta n_p \]

\[ I_p(x_n=0) = qA \frac{D_p}{L_p} \Delta p_n \]

Slopes of minority carrier concentrations.

\[ I = I_p(x_n=0) - I_n(x_p=0) = qA \left( \frac{D_p}{L_p} \Delta p_n + \frac{D_n}{L_n} \Delta n_p \right) \]

\[ = qA \left( \frac{D_p \Delta p_n}{L_p} + \frac{D_n \Delta n_p}{L_n} \right) \left( e^{qV/kT} - 1 \right) \]

Steady state charge storage.

\[ Q_n = -qA \int \delta n(x_p) \, dx_p \]

\[ Q_p = qA \int \delta p(x_n) \, dx_n \]

\[ I_n(x_p=0) = \frac{Q_n}{\tau_n} = -qA \frac{L_n}{\tau_n} \Delta n_p \]

\[ I_p(x_n=0) = \frac{Q_p}{\tau_p} = \frac{qA L_p}{\tau_p} \Delta p_n \]
Minority carrier drift can be ignored outside of the space charge region, but what about the majority carrier contribution to the current?

Total current is conserved...

\[ I_{\text{total}} = I_{\text{majority}} - I_{\text{minority}} \]
Is that the whole story?

- Far from the junction the current is carried all by electrons.

- Electrons must flow in from the n material to resupply the electrons lost by recombination in the excess hole distribution.

The electron current includes sufficient electron flow to supply the recombination at $x_n$ and the injection of electrons into the p-material.

Flow of electrons in the n-material towards the junction forms a current in the +x direction.
But is this current drift, or diffusion or both?

Near the junction the majority carrier concentration changes with the minority carrier concentration to keep the device charge neutral.

- In the bulk most of the current is drift as there are no gradients in the concentrations.

- As we approach the junction, carrier concentrations change and we get a combination of drift and diffusion. Drift will dominate for majority carriers.

- Note that the electric field in the neutral regions cannot be zero, as we assumed but since, we have a large majority carrier concentration, the field need not be large.
Reverse Bias

Most of the preceding analysis dealt with forward bias, what about the reverse bias case?

We can use the same equations and analysis to determine the reverse bias behavior...

Set \( V = -V_r \) which biases the p-side negatively with respect to the n-side and examine the relationship for the excess hole concentration...

\[
\Delta p_n = p(x_{n0}) - p_n = p_n(e^{qV_r/kT} - 1)
\]

\[
\Delta p_n = p_n(e^{q(-V_r)/kT} - 1) \approx -p_n \quad V_r >> k_b T/q
\]

• For large reverse bias, the minority carrier concentration goes to zero.
• Minority carrier concentration equations still given by previously derived equations.
• Depletion of minority carriers extends one diffusion length on either side of the junctions.
• Referred to as **minority carrier extraction**.
Reverse Bias

What is happening physically to the carriers...

• Carriers are being swept down the barrier at the junction to the other side.

• They are not being replaced by an opposing diffusion of carriers.

• Reverse bias saturation occurs because of drift of carriers down the barrier.

• But the rate of drift depends on the rate of minority carriers arrive by diffusion from the neutral material supplied by thermal generation.
And the quasi-Fermi levels move again...

- $F_n$ moves farther away from EC towards EV because in reverse bias we have fewer carriers than in equilibrium.
- Quasi-Fermi levels here go inside the bands but we need to remember that $F_p$ is a measure of the hole concentration and is correlated with EV and not EC.
- This just tells us we have very few holes (smaller than in equilibrium).