ECE 340
Lecture 21: P-N Junction II

Class Outline:
- Contact Potential
- Equilibrium Fermi Levels
Key Questions

• What is the contact potential?
• Where does the transition region come from?
• How can I understand the contact potential quantitatively?
• What is the depletion approximation?
Now let’s start analyzing the p-n junction...

We want to form a compromise between a very detailed description of the physics of the p-n junction and a solid qualitative understanding of its operation...

We have two different models:

1. Step junction (alloyed and epitaxial junctions)

2. Graded junctions (diffused junctions where \( N_d - N_a \) varies over a significant distance across the junction)

Our plan: Explore the step junction and extend the understanding to deal with the graded junction.

Our starting point: We are starting in equilibrium with no external excitation and no external currents.
Contact Potential

What do we expect to happen when we join them together?

We expect to find the following when we connect the p-type semiconductor with the n-type semiconductor:

• The potential difference between the two regions resulting from the doping discrepancy will cause a potential difference between the two regions.

• There should be four components of current across the junction due to drift and diffusion.

• The four components must add to give zero net current at equilibrium.
Contact Potential

But we already know what will happen when we join them together...

\[ J_p^{(drift)} + J_p^{(diff)} = 0 \]
\[ J_n^{(drift)} + J_n^{(diff)} = 0 \]
Contact Potential

So what is the **contact potential**...

The contact potential is the voltage necessary to maintain equilibrium at the junction.

The contact potential does not imply the presence of an external potential.

The contact potential cannot be measured because when contact is made to the junction a potential forms at the contacts which works to cancel the contact potential.

The contact potential also separates the bands with the conduction energy bands higher on the p-side. This is required to make the Fermi level constant.
Contact Potential

Let's begin to develop a quantitative relationship for the \textit{contact potential}...

Let's start by developing a relationship between the \textit{contact potential} \( (V_0) \) and the doping concentrations on either side of the junction...

\[
J_n(x) = q\mu_n n(x)E(x) + qD_n \frac{dn(x)}{dx} \\
J_p(x) = q\mu_p p(x)E(x) - qD_p \frac{dp(x)}{dx}
\]

Begin with the current densities

Focus for the moment on the hole current. In equilibrium, the different components should cancel.

\[
J_p(x) = q\mu_p p(x)E(x) - qD_p \frac{dp(x)}{dx} = 0
\]
Contact Potential

Now begin to simplify the equation...

\[
\frac{\mu_p}{D_p} \varepsilon(x) = \frac{1}{p(x)} \frac{dp(x)}{dx} \quad \rightarrow \quad \varepsilon(x) = -\frac{d\varphi(x)}{dx}
\]

Relate the electric field in the above equation to the gradient in the potential, and apply the Einstein relations. After we arrive at:

\[
-\frac{q}{kT} \frac{d\varphi(x)}{dx} = \frac{1}{p(x)} \frac{dp(x)}{dx}
\]

This is a simple differential equation we can solve by integration. But what are the limits?

What we want:
1. Potential on either side of the junction \( \varphi_N \) and \( \varphi_P \).
2. Hole concentration on either side of the junction.

Einstein Relations

\[
\begin{align*}
D_N &= \frac{k_b T}{q} \\
\mu_N &= \frac{q}{k_b T} \\
D_P &= \frac{k_b T}{q} \\
\mu_P &= \frac{q}{k_b T}
\end{align*}
\]
Contact Potential

Now we assume that we are dealing with a step-potential...

In doing so we can further assume that the electron and hole concentrations outside of the transition region are at their equilibrium values. So let’s integrate...

\[- \frac{q}{kT} \int_{\psi_p}^{\psi_n} d\psi = \int_{p_p}^{p_n} \frac{1}{p} dp\]

Finish representing the contact potential by relating it to the hole (or electron) concentrations on either side of the junction...

\[- \frac{q}{kT} (\psi_n - \psi_p) = \ln p_n - \ln p_p = \ln \frac{p_n}{p_p}\]

But this may not be the most useful as we are often dealing with extrinsic semiconductors, so let’s rewrite it...

\[V_0 = \frac{kT}{q} \ln \frac{p_p}{p_n}\]

\[V_0 = \frac{kT}{q} \ln \frac{N_a}{n_i^2/N_d} = \frac{kT}{q} \ln \frac{N_a N_d}{n_i^2}\]
Contact Potential

By assuming the majority carrier concentration to be the doping, we can find another form of the equation...

\[
\frac{p_p}{p_n} = e^{\frac{qV_0}{kT}}
\]

Using the equilibrium carrier concentration equations, we can extend the above equation to include the electron concentrations on either side of the junction...

\[
p_p n_p = n_i^2 = p_n n_n
\]

\[
\frac{p_p}{p_n} = \frac{n_n}{n_p} = e^{\frac{qV_0}{kT}}
\]

Will be useful later when we apply bias.
We are in equilibrium, so is this consistent with the definitions for the contact potential that we have just derived?

Let’s redefine the contact potential in terms of what we know about the equilibrium carrier concentrations that we assume to prevail far from the transition region...

\[
p_p = e^{qV_0/kT} \quad \frac{p_p}{p_n} = e^{qV_0/kT}
\]

• Energy bands are separated by the contact potential.

• Fermi levels on either side of the junction are equal in equilibrium.

• When we apply bias, the contact potential is lowered and the Fermi levels change.
Depletion Approximation

To gain a qualitative understanding of the solution for the electrostatic variables we need Poisson’s equation:

\[ \frac{d\varphi}{dx} = \frac{\rho}{K_S \varepsilon_0} = \frac{q}{K_S \varepsilon_0} (p - n + \frac{N_D - N_A}{\varepsilon_0}) \]

Most times a simple closed form solution will not be possible, so we need an approximation from which we can derive other relations. Consider the following:

- To obtain the electric field and potential we need to integrate.
- However, we don’t know the electron and hole concentrations as a function of x.
- Electron and hole concentrations are a function of the potential which we do not know until we solve Poisson’s equation.

Use the **depletion approximation**...
**Depletion Approximation**

We already have the foundation for this approximation...

- A prominent feature in the qualitative solution was the appearance of a nonzero charge density straddling the junction.

- The charge arises because the carrier numbers are reduced by diffusion across the junction.

- The carrier depletion tends to be greatest in the immediate vicinity of the junction.
Depletion Approximation

What does the **depletion approximation** tell us...

1. The carrier concentrations are assumed to be negligible compared to the net doping concentrations in the junction region.

2. The charge density outside the depletion region is taken to be identically zero.

Poisson equation becomes...

\[
\frac{d\varepsilon}{dx} \approx \begin{cases} 
\frac{q}{K_S \varepsilon_0} (N_D - N_A) & \ldots -x_p \leq x \leq x_n \\
0 & \ldots x \leq -x_p \text{ and } x \geq x_n
\end{cases}
\]
Depletion Approximation

What does the **depletion approximation** look like...
Let’s solve a few problems:

A silicon step junction is maintained at room temperature with doping concentrations such that $E_F = E_V - 2kT$ on the p-side and $E_F = E_C - E_G/4$ on the n-side.

(a) Draw the band diagram
(b) Determine the contact potential

Consider the p1-p2 isotope junction shown here:

(a) Draw the band diagram for the junction taking the doping to be non-degenerate and $N_{A1} > N_{A2}$.
(b) Derive an expression for the contact potential.
(c) Make rough sketches of the electric field, potential and charge density.