ECE 340
Lecture 11: Effects of Temperature and High Field on Mobility

Class Outline:

• Drift and Resistance
• Temperature and Mobility
• High Field Effects
• Band Bending
Key Questions

- What is the resistivity?
- How does temperature effect the mobility and resistivity?
- How does high field effect carrier drift?
- What is band bending and how does it relate to carriers?
**Drift and Resistance**

When we apply an electric field, *carriers begin to drift*...

We can define a drift velocity:

\[
\langle v_x \rangle = \frac{\langle p_x \rangle}{m_{n,p}^*} = \pm \frac{q \tau_c E_x}{m_{n,p}^*}
\]

The directed electric field that we apply to our semiconductor leads to a current flow...

*Which resulted in electron and hole mobilities...*
Drift and Resistance

We were able to obtain the **resistivity and the conductivity**...

We start with the **total drift current**:

\[ J_{\text{drift}} = J_{\text{drift}}^n + J_{\text{drift}}^p \]

\[ \mathbf{E} \]

\[ J_{\text{drift}}^n \quad J_{\text{drift}}^p \]

Plug in what we know about mobilities...

\[ J_{\text{drift}} = q(n\mu_n + p\mu_p)E \]

Then relate it back to Ohm's law...

\[ J = \sigma E = \frac{E}{\rho} \quad \sigma = \text{conductivity} \left[ \frac{1}{\Omega \cdot cm} \right] \]

\[ \rho = \frac{1}{\sigma} = \frac{1}{q(n\mu_n + p\mu_p)} \quad \rho = \text{resistivity} \left[ \Omega \cdot cm \right] \]

• Resistivity and conductivity are inverses of one another.
• **Resistivity measures a material's inherent resistance to current flow.**
Drift and Resistance

We can use the **resistivity** to specify the doping...

We know the expression for the resistivity when both carrier types are contributing:

\[
\rho = \frac{1}{\sigma} = \frac{1}{q(n\mu_n + p\mu_p)}
\]

But for **extrinsic** materials, we can simplify:

\[
\rho = \frac{1}{q\mu_n N_D} \quad N_D \gg n_i
\]

\[
\rho = \frac{1}{q\mu_n N_A} \quad N_A \gg n_i
\]

When combined with the mobility versus doping data, we can determine the doping concentrations.
Drift and Resistance

Let’s try to visualize what is going on…

Carrier motion in a material:

For this structure, we can use the dimensions to define the resistance:

\[ R = \frac{\rho L}{wt} = \frac{L}{\rho wt} \]

• Carriers move in a group.
  • Electrons move against the electric field.
  • Holes move with the electric field.

• Convention makes the drift currents flow in the same direction.

• The contacts are considered ohmic
  • Perfect sources and sinks for carriers.
  • No tendency to inject either carrier.

Electrons are easy to visualize.

What happens to holes when they reach the contact?
Drift and Resistance

What happens to the holes??

- As the hole reaches the end of the semiconductor, it recombines with an electron which must be supplied by the external circuit.
- As one hole disappears, another hole must appear at the entrance of the circuit to conserve charge neutrality.
- So, we have the generation of an electron-hole pair when an electron leaves the semiconductor sample.
- The hole flows in while the electron flows out.
Drift and Resistance

Let’s do a simple example...

Consider a Si sample doped with $10^{16}$ cm$^{-3}$ Boron. What is the resistivity?

$N_A = 10^{16}$ cm$^{-3}$ \hspace{1cm} $N_D = 0$

From the Boltzmann relations:

$p \sim 10^{16}$ cm$^{-3}$ and $n \sim 10^4$ cm$^{-3}$

Use the simplified resistivity relation:

$$\rho = \frac{1}{q\mu_p N_A}$$

$$\rho = 1.4 \Omega \cdot cm$$
Drift and Resistance

What if I take the same sample and add $10^{17}$ cm$^{-3}$ Arsenic?

$N_A = 10^{16}$ cm$^{-3}$  \hspace{0.5cm}  N_D = 10^{17}$ cm$^{-3}$

From the Boltzmann relations:

$n \sim 9 \times 10^{16}$ cm$^{-3}$ and $p \sim 1.1 \times 10^3$ cm$^{-3}$

Use the simplified resistivity relation:

$$\rho = \frac{1}{q\mu_n N_D}$$

$$\rho = 0.092 \Omega \cdot cm$$
Drift and Resistance

Yet another example...

Consider another silicon bar...

L = 0.1 cm

A = 100 µm²

N_D = 10^{17} cm^{-3}

Find the current with 10 V applied at 300K.

For this doping, \( \mu_n = 750 \text{ cm}^2/\text{Vs} \)

\[
\rho = \frac{1}{q \mu_n n} = \frac{1}{(1.6 \times 10^{-19})(750)(10^{17})} = 0.083 \Omega \cdot \text{cm}
\]

\[
R = \frac{\rho L}{A} = 8333k\Omega \quad \Rightarrow \quad I = \frac{V}{R} = 1.2mA
\]
Example: I want to construct a temperature sensor for measuring the ambient temperature. How can I do it?
Temperature and Mobility

What does temperature mean to mobility?

Silicon mobility at 300 K

Lattice Scattering
Ionized Impurity Scattering
Temperature and Mobility

Seems like there are two main competing phenomena...

**Impurity (dopant) Scattering:**

- The force acting on the particles is Coulombic.
- There is less change in the electron’s direction of travel the faster it goes.

\[ \mu_{\text{impurity}} \propto \frac{V_{th}^3}{N_A + N_D} \propto \frac{T^2}{N_A + N_D} \]

**Phonon (lattice) Scattering:**

\[ \mu_{\text{phonon}} = \frac{q \tau_{\text{phonon}}}{m^*} \propto \frac{1}{\#_{\text{phonons}} \times V_{th}} \propto \frac{1}{T \times T^{1/2}} \propto T^{-3/2} \]

Phonon scattering mobility decreases as the temperature increases.

Since the scattering probability is inversely proportional to the mean free time and the mobility, we can add individual scattering mechanisms inversely.

\[ \frac{1}{\mu} = \frac{1}{\mu_{\text{phonon}}} + \frac{1}{\mu_{\text{impurity}}} \]
Temperature and Mobility

Let’s look at how these effects interplay in Si...

Electrons

Holes

Temperature $T$ (K)

Mobility $\mu$ (cm$^2$ V$^{-1}$ s$^{-1}$)

Impurity scattering
Lattice scattering

$T^3/2$
$T^{-3/2}$
High Field Effects

Strange things happen at high fields...

- The entire time we’ve assumed that Ohm’s law is valid in the drift process.
- At high fields, we can get a sub-linear dependence in the drift velocity.
- Carriers at high fields are referred to as “hot carriers”
- **When the carriers reach the thermal velocity limit, the additional energy imparted to the system is all given to the lattice.**
- The result is scattering limited velocity.
- Some materials are weirder than others (GaAs).
Fermi Level Invariance

Question: Is it in equilibrium?

- Assume two materials in intimate contact.
- In thermal equilibrium.
  - No current.
  - No net energy transfer.
- Carriers moving from 2 to 1 must be balanced by carriers moving from 1 to 2.

Material 1
DOS – $N_1(E)$
FD – $f_1(E)$

Material 2
DOS – $N_2(E)$
FD – $f_2(E)$

Energy

\[ \begin{align*}
\text{Rate}_{1-2} & \Rightarrow N_1(E)f_1(E) \cdot N_2(E)[1 - f_2(E)] \\
\text{Rate}_{2-1} & \Rightarrow N_2(E)f_2(E) \cdot N_1(E)[1 - f_1(E)] \\
\text{Rate}_{1-2} & = \text{Rate}_{2-1}
\end{align*} \]

Therefore...
\[ f_1(E) = f_2(E) \]
\[ E_{f1} = E_{f2} \]
\[ \frac{dE_F}{dx} = 0 \]

YES
Band Bending

The energy bands are not constant in fields!

Instead, they move as a function of position.

- We need energy equal to the band gap to break bonds and excite carriers to the conduction band.
- If we only impart enough energy to promote an electron then it simply sits in the conduction band.
- Extra energy allows carriers to move.

Total Electron Energy

T.E. = K.E. + P.E.

Electron Kinetic Energy (E - E_c)

Potential Energy = E_c - E_{ref}

Hole Kinetic Energy (E_v - E)

P.E.

E_{ref}
Band Bending

We can use elementary physics to determine the potential energy...

\[ \text{Potential Energy} = E_c - E_{\text{ref}} \]

Total Electron Energy

Electron Kinetic Energy \((E - E_c)\)

Hole Kinetic Energy \((E_v - E)\)

\[ \text{T.E.} = \text{K.E.} + \text{P.E.} \]

The potential energy:

\[ -qV \]

But previously we said:

\[ E_c - E_{\text{ref}} \]

Electrostatic potential
But we can still determine more...

Total Electron Energy

\[ \text{T.E.} = \text{K.E.} + \text{P.E.} \]

Electron Kinetic Energy \((E - E_c)\)

Hole Kinetic Energy \((E_v - E)\)

Potential Energy \(E_c - E_{\text{ref}}\)

By definition, we know:

\[
E = -\nabla V = \frac{1}{q} \frac{dE_c}{dx} = \frac{1}{q} \frac{dE_v}{dx} = \frac{1}{q} \frac{dE_i}{dx}
\]