

Gate controlled spin-density wave and chiral FFLO superconducting phases in interacting helical liquids

Qinglei Meng,¹ Taylor L. Hughes,¹ Matthew J. Gilbert,^{2,3} and Smitha Vishveshwara¹

¹*Department of Physics, University of Illinois, 1110 West Green Street, Urbana, Illinois 61801, USA*

²*Department of Electrical and Computer Engineering, University of Illinois, 1406 West Green Street, Urbana, Illinois 61801, USA*

³*Micro and Nanotechnology Laboratory, University of Illinois, 208 N. Wright Street, Urbana, Illinois 61801, USA*

(Received 4 December 2011; revised manuscript received 17 July 2012; published 8 October 2012)

We explore the phases exhibited by an interacting helical liquid in the presence of finite chemical potential (applied gate voltage) and spin imbalance (applied magnetic field). We find that the helical nature gives rise to quantum orders that are expected to be absent in nonchiral one-dimensional electronic systems. For repulsive interactions, the ordered state has an oscillatory spin texture whose ordering wave vector is controlled by the chemical potential. We analyze the manner in which a magnetic impurity provides signatures of such oscillations. We find that finite spin imbalance favors a finite current carrying ground state that is not condensed in the absence of interactions and is superconducting for attractive interactions. This state is characterized by Fulde-Ferrell-Larkin-Ovchinnikov (FFLO)-type oscillations where the Cooper pairs obtain a finite center-of-mass momentum. These phases can be realized on the edge of two-dimensional systems exhibiting the quantum spin Hall effect or on dislocation lines in weak topological insulators.

DOI: [10.1103/PhysRevB.86.155110](https://doi.org/10.1103/PhysRevB.86.155110)

PACS number(s): 75.30.Hx, 71.55.-i, 72.25.Hg, 74.20.Mn

The quantum spin Hall (QSH)/two-dimensional (2D) time-reversal invariant topological insulator state¹⁻⁵ has attracted much interest since its recent discovery. One reason it remains an important topic beyond its realization in HgTe quantum wells is due to the prediction of similar a new universality class of interacting 1D liquids: the helical liquid.^{6,7} The helical liquid consists of an odd number of counterpropagating Kramers' pairs and remains metallic, even when disorder is present, as long as time-reversal symmetry is preserved. This helical liquid exhibits unusual features such as fractional charge, Kramers' pairs of Majorana bound states, and individual Majorana bound states when in the proximity of magnets, superconductivity, or both, respectively.⁸⁻¹⁰

Beyond the robustness to disorder, the fate of the helical liquid in the presence of interactions is of interest. Interactions provide a new type of 1D interacting system that can be realized in experiments in HgTe/CdTe, InAs/GaSb quantum wells or in dislocation lines in weak topological insulators.^{5,11-13} The helical liquid is intrinsically different from a conventional 1D electron gas (1DEG) as the direction of propagation is correlated with the direction of the spin polarization which reduces the degrees of freedom by half. In this work, we explore the rich 1D phases intrinsic to the helical liquid in the presence of interactions. We observe that repulsive interactions can lead to a spin-density-wave phase that is unique to the helical liquid and argue that it is not generated in a typical 1DEG. On the other hand, attractive interactions render the liquid unstable to the formation of a Fulde-Ferrell-Larkin-Ovchinnikov (FFLO)-type superconducting phase,^{14,15} which is an *s*-wave-like order parameter that condenses at a finite wave vector. The nature of the helical liquid gives it an upper hand for hosting the FFLO-type phase, which has eluded experimental observation in higher dimensions.¹⁶⁻¹⁸ The two phases emerging from the repulsive versus attractive regimes are dual to each other in that the roles of spin and charge are exchanged. Finally, we show that with repulsive interactions a magnetic impurity acts as an effective experimental probe

of the QSH edge, inducing oscillations in the magnetization *direction* which are fundamentally different from the Ruderman-Kittel-Kasuya-Yosida (RKKY) oscillations of the magnetization *amplitude* in a conventional 1DEG.¹⁹⁻²¹

We begin with a heuristic analysis of the noninteracting helical edge state in the presence of finite chemical potential and spin imbalance, focusing on the fundamental differences that give rise to new order when compared with a typical 1DEG. As shown in Fig. 1(a), the helical edge state consists of linearly dispersing spin-dependent modes associated with a Dirac point centered at zero momentum, and is described by the Hamiltonian

$$H_0 = v \int dx [\psi_{R\uparrow}^\dagger(x)(-i\partial_x)\psi_{R\uparrow}(x) - \psi_{L\downarrow}^\dagger(x)(-i\partial_x)\psi_{L\downarrow}(x)], \quad (1)$$

where v is the velocity and x is the coordinate tangent to the edge of the sample. The operator $\psi_{R\uparrow(L\downarrow)}(x)$ annihilates electron moving to the right (left) with up (down) spin at position x . The effects of a nonzero chemical potential and a spin imbalance can be described by

$$H_\mu = \int dx [-\mu_\uparrow \psi_{R\uparrow}^\dagger(x)\psi_{R\uparrow}(x) - \mu_\downarrow \psi_{L\downarrow}^\dagger(x)\psi_{L\downarrow}(x)], \quad (2)$$

where $\mu_{\uparrow(\downarrow)}$ is an effective chemical potential for up (down) spin in the helical liquid. The chemical potential, $\mu = \frac{1}{2}(\mu_\uparrow + \mu_\downarrow)$, can be controlled by tuning a gate voltage, and the spin imbalance, $\delta_S = \mu_\uparrow - \mu_\downarrow$, may be controlled by applying magnetic field in the direction perpendicular to the transport plane (or more generally, parallel to the direction of spin polarization of the edge state). In fact, because of the spin-momentum locking on the edge, a *spin* imbalance acts to give rise to a *charge* current.

Given the fundamental fields comprising the helical edge state, the two lowest-order operators that could develop

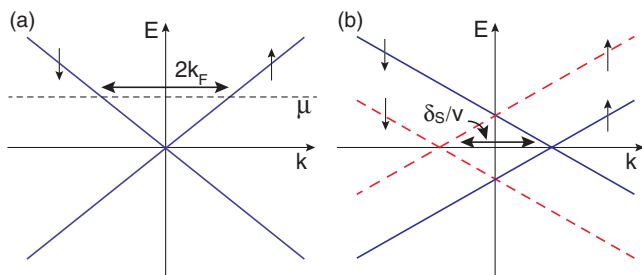


FIG. 1. (Color online) Noninteracting picture for spin-density wave and chiral FFLO-type superconductor state formation. (a) Edge state energy spectra at chemical potential μ . It is energetically favorable to open a gap at the Fermi level as opposed to the Dirac point, thus forming a spin-density wave with wave vector $2k_F$. (b) Bogoliubov-de Gennes spectrum for nonzero δ_S . Solid lines are electron states, dashed lines are hole states. Hybridization must occur between a solid and dashed line with opposite spin leading to a finite pairing wave vector of δ_S/v .

nonvanishing expectation values in an ordered phase are

$$O_m = \psi_{R\uparrow}^\dagger(x)\psi_{L\downarrow}(x), \quad O_\Delta = \psi_{R\uparrow}(x)\psi_{L\downarrow}(x). \quad (3)$$

These order parameters represent magnetic order ($\langle O_m \rangle$) and superconducting order ($\langle O_\Delta \rangle$) and are dual to one another with regard to charge and spin in that the former carries charge 0 and spin $2\hbar/2$ while $\langle O_m \rangle$ carries charge $2e$ and spin 0.

We now argue that for nonvanishing μ and δ_S , these order parameters have the property that they are inhomogeneous in space, exhibiting oscillatory behavior over a characteristic length scale. We begin by tuning $\mu = \delta_S = 0$ and considering magnetic order. The system is tuned to the Dirac point and any ferromagnetic order perpendicular to the spin polarization of the edge states would open a gap at $k = 0$ since it would couple via a constant multiplying a Pauli spin matrix. If one tunes μ away from zero then, in order to open a gap at the Fermi level, the magnetic order must have a finite wave vector of $q_m^{(0)} \equiv -2\mu/v = -2k_F$ where the superscript refers to the free limit [see Fig. 1(a)]. Thus we induce a spin-density wave so that a gap can open at the Fermi level as opposed to a gap opening at the (buried) Dirac point for ferromagnetic ordering. This type of chemical potential driven spin-density wave is unique to the helical liquid as seen by noting the form of a magnetic order parameter for a full 1DEG:

$$\begin{aligned} \psi_\uparrow^\dagger(x)\psi_\downarrow(x) &\sim (e^{-ik_F x}\psi_{R\uparrow}^\dagger(x) + e^{ik_F x}\psi_{L\uparrow}^\dagger(x)) \\ &\quad \times (e^{ik_F x}\psi_{R\downarrow}(x) + e^{-ik_F x}\psi_{L\downarrow}(x)) \\ &= \psi_{R\uparrow}^\dagger\psi_{R\downarrow} + \psi_{L\uparrow}^\dagger\psi_{L\downarrow} + (e^{-2ik_F x}\psi_{R\uparrow}^\dagger\psi_{L\downarrow} + \text{c.c.}). \end{aligned}$$

For the full 1DEG the nonscillatory terms generically dominate, but these terms are completely absent for the helical liquid which only has $e^{-2ik_F x}\psi_{R\uparrow}^\dagger\psi_{L\downarrow}$ nonvanishing. Thus the existence of a spin-density wave is a unique signature of the reduced degrees of freedom of the helical liquid as compared to a conventional IDEG.

Now let us consider the effects of a nonzero δ_S in the noninteracting limit for which we will return to the free-fermion Hamiltonian. In the Bogoliubov-de Gennes formalism the

Hamiltonian can be rewritten

$$H_{\text{BdG}} = \int dx \Psi^\dagger(x) [-iv\partial_x \mathbb{I} \otimes \sigma^z - (\delta_S/2)\tau^z \otimes \sigma^z] \Psi(x),$$

where τ^a represents particle-hole space and σ^a spin space, and $\Psi(x) = [\psi_{R\uparrow}(x) \psi_{L\downarrow}(x) \psi_{R\uparrow}^\dagger(x) \psi_{L\downarrow}^\dagger(x)]^T$. This has energy levels $E_\pm = \pm|vk| \pm \delta_S/2$ (with uncorrelated signs). A homogenous s -wave pairing cannot open a gap at the Fermi level if $\delta_S \neq 0$ and is thus energetically frustrated. As shown in Fig. 1(b) the pairing term must have a finite wave vector $q_\Delta^{(0)} \equiv \delta_S/v$ in the noninteracting limit to open a gap. A full 1DEG would have both $\psi_{R\uparrow}^\dagger\psi_{L\downarrow}^\dagger + \psi_{R\downarrow}^\dagger\psi_{L\uparrow}^\dagger$ pairing terms while the QSH edge only has the former. Thus in the helical case there is always a ground-state current of Cooper pairs in one direction picked by the sign of δ_S since the order parameter oscillates like $e^{iq_\Delta x}$ instead of $\cos(q_\Delta x)$, as in fact originally considered by Fulde and Ferrell.¹⁴ We refer to this state as a chiral FFLO state.

We now turn to the effects of interactions and their crucial role in determining the fate of the helical edge state and energetically favorable ordered states. We derive the corresponding phase diagram by analyzing the form of the susceptibilities associated with each order and show, as might be expected, that magnetic (superconducting) order is stabilized by repulsive (attractive) interactions. We note that we are only considering equilibrium states. As in previous treatments,^{6,7,22} we ignore umklapp scattering and employ the following form for interactions between edge electrons:

$$\begin{aligned} H_I &= \frac{g_4}{2} \int dx \psi_{R\uparrow}^\dagger(x)\psi_{R\uparrow}(x)\psi_{R\uparrow}^\dagger(x)\psi_{R\uparrow}(x) \\ &\quad + \frac{g_4}{2} \int dx \psi_{L\downarrow}^\dagger(x)\psi_{L\downarrow}(x)\psi_{L\downarrow}^\dagger(x)\psi_{L\downarrow}(x) \\ &\quad + g_2 \int dx \psi_{R\uparrow}^\dagger(x)\psi_{R\uparrow}(x)\psi_{L\downarrow}^\dagger(x)\psi_{L\downarrow}(x), \end{aligned} \quad (4)$$

where $g_{2(4)}$ represents the forward scattering strength of different (identical) species. These terms come directly from short-range density-density interactions and have been extensively studied in Refs. 6 and 7. As done previously,^{6,7,22} the interacting system can be explored within a Luttinger liquid framework by representing the fermion fields in terms of boson fields ϕ and θ : $\psi_{R\uparrow}(x) \sim e^{-i[\phi(x)-\theta(x)]}$, $\psi_{L\downarrow}(x) \sim e^{i[\phi(x)+\theta(x)]}$. Thus the interacting helical liquid described by $H = H_0 + H_\mu + H_I$ is mapped into a free boson gas with a Hamiltonian

$$H = \frac{1}{2\pi} \int dx \left[uK(\nabla\theta)^2 + \frac{u}{K}(\nabla\phi)^2 + 2\mu\nabla\phi - \delta_S\nabla\theta \right], \quad (5)$$

where $u = v[(1 + \frac{g_4}{2\pi v})^2 - (\frac{g_2}{2\pi v})^2]^{1/2}$ is the renormalized velocity and $K = (\frac{1 + \frac{g_4}{2\pi v} - \frac{g_2}{2\pi v}}{1 + \frac{g_4}{2\pi v} + \frac{g_2}{2\pi v}})^{1/2}$ is the Luttinger parameter. Values of $K < (>)$ 1 represent repulsive (attractive) interactions. The chemical potential terms μ and δ_S can be absorbed as inhomogeneous shifts of the bosonic fields,

$$\tilde{\phi}(x) = \phi(x) + \mu Kx/u, \quad \tilde{\theta}(x) = \theta(x) - \delta_S x/2Ku, \quad (6)$$

which transforms the Hamiltonian to the standard form $H = \frac{1}{2\pi} \int dx [uK(\nabla\tilde{\theta})^2 + \frac{u}{K}(\nabla\tilde{\phi})^2]$. Thus while the QSH system

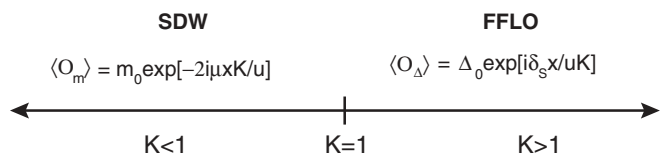


FIG. 2. $T = 0$ phase diagram of QSH edge for interactions characterized by K , finite chemical potential μ , and finite spin imbalance δ_S .

bears key differences in the physics, at the technical level, several of its properties can be mapped to the extensively analyzed Luttinger liquid system describing the low-energy physics of a *spinless* interacting 1DEG.

From Eq. (6), it immediately follows that the magnetic and superconducting orders are both associated with oscillations that are renormalized by the interactions. By noting that $O_m \sim e^{2i\phi(x)}$, $O_\Delta \sim e^{2i\theta(x)}$ and using the shifted forms $O_m \sim e^{-2i\mu Kx/u} \tilde{O}_m$, $O_\Delta \sim e^{i\delta_S x/(uK)} \tilde{O}_m$, we conclude that

$$\langle O_m \rangle = m_0 \exp[iq_m x], \quad \langle O_\Delta \rangle = \Delta_0 \exp[iq_\Delta x], \quad (7)$$

where $q_m = -2\mu K/u$ and $q_\Delta = \delta_S/uK$. To determine which of the orders dominates, we inspect the form of the associated susceptibilities, given by $\chi_{m/\Delta}(x, \tau) = -(T_\tau O_{m/\Delta}(x, \tau) O_{m/\Delta}^\dagger(0, 0))$, where τ is imaginary time. We adapt the standard Luttinger liquid treatment²³ to our situation to obtain the following temperature dependence in the Fourier domain:

$$\begin{aligned} \chi_m(k = q_m, \omega = 0) &\sim T^{2K-2}, \\ \chi_\Delta(k = q_\Delta, \omega = 0) &\sim T^{(2K-1-2)}. \end{aligned} \quad (8)$$

The finite wave-vector dependence reflects the oscillatory behavior in Eq. (7) and the stability of a particular order is indicated by the divergence of the associated susceptibility for $T \rightarrow 0$, as summarized in the phase diagram of Fig. 2. Hence for repulsive interactions, $K < 1$, the system magnetically orders and is characterized by oscillations whose wave vector q_m is controlled by the applied chemical potential. For attractive interactions, $K > 1$, the system tends to form a superconducting state that shows chiral FFLO-type oscillations having the feature that the wave vector q_Δ is completely tunable via an applied spin imbalance.

Given that the currently available QSH systems are all in the repulsively interacting regime, we now focus on probing the magnetic phase associated with $K < 1$. We show that a weak, localized magnetic impurity that provides an in-plane magnetic field $\mathbf{H}(x) = \mathbf{H}\delta(x)$ acts as the simplest means of observing the oscillations in the magnetic order of Eq. (7). As we seek to probe the response of the system by tunneling into the helical liquid, we must understand how this coupling affects our system. The tunnel coupling into the edge liquid due to such a magnetic perturbation is given by

$$\begin{aligned} H_H &= -\mu_B \psi^\dagger (\sigma_x H_x + \sigma_y H_y) \delta(x) \psi, \\ &= -\mu_B |\mathbf{H}| [O_m(x) e^{-i\xi} + O_m^\dagger(x) e^{i\xi}] \delta(x), \end{aligned}$$

where μ_B is the Bohr magneton, $\psi = (\psi_{R\uparrow} \ \psi_{L\downarrow})^T$ and $\xi = \tan^{-1}(H_y/H_x)$. As shown in Fig. 3(a), a spin-up electron impinging the impurity effectively backscatters into a

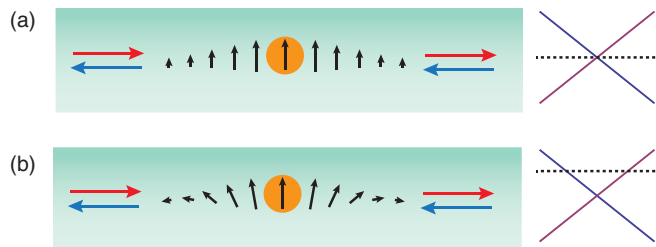


FIG. 3. (Color online) Magnetization oscillations around a single-magnetic impurity on a quantum spin Hall edge which decay as one moves away from the impurity. (a) $\mu = 0$ in the weakly repulsive regime leads to a ferromagnetic domain, (b) $\mu > 0$ in the weakly repulsive regime leads to a domain with oscillatory magnetization direction.

spin-down electron and vice versa. The magnetic perturbation, upon suppressing the spin indices in the $(\psi_{R\uparrow} \ \psi_{L\downarrow})$ fields, exactly maps to the well-known quantum impurity problem in spinless quantum wires whose scaling properties can be easily analyzed within the Luttinger liquid framework.²³ In fact, the response to the impurity in our situation parallels the features of Friedel oscillations in the vicinity of a nonmagnetic impurity in a spinless Luttinger.²⁴ At high energies and short distances, set by the bare magnetic impurity strength, the impurity can be treated perturbatively. Meanwhile at low energies and large distances, interactions renormalize its strength and the behavior is governed by the strong-coupling fixed point wherein the impurity effectively splits the system into two pieces. The resultant magnetization in the helical liquid $m_+(x, t) \equiv m_x(x, t) + im_y(x, t) = 2\mu_B \langle O_m(x, t) \rangle$ takes the form

$$m_+(x) = \frac{\mu_B}{\pi\alpha} e^{i(q_m x + \xi)} f(x, T, K, |\mathbf{H}|), \quad (9)$$

where α is a short-distance cutoff determined by the bulk energy gap, and f is a dimensionless decaying envelope function whose form depends on the regime being probed by the tunneling.²⁴ For instance, in the perturbative regime where the temperature is high or the distance being probed is close to the impurity, the susceptibility of the impurity-free system χ_m determines the response to the local impurity. For $T \rightarrow 0$, this gives $f \sim x^{1-2K}$ for $\alpha \ll x \ll x_0$, where x_0 is a characteristic scale set by the bare impurity strength. On the other hand, for $x \gg x_0$, which corresponds to the probe distances far from the impurity or at low temperatures, the strong coupling analysis gives $f \sim x^{-K}$. For a more general form of magnetic quantum impurity coupling, the helical liquid shows a rich range of behavior, including modified Kondo physics^{25,26} which, in this context, necessitates an investigation of the finite gate-potential induced spin-density-wave (SDW) physics.

Regardless of the strengths of the impurity and interactions, and the regimes being probed, the ubiquitous feature of the magnetization is the $2q_m$ dependence in Eq. (9) that reflects SDW ordering. As illustrated in Fig. 3, the impurity thus creates oscillations in the magnetization which decay with distance. More explicitly, if, for example, $H_x \neq 0$ and $H_y = 0$, we have $(m_x, m_y) \sim [\cos(2\frac{K}{u}\mu x), -\sin(2\frac{K}{u}\mu x)]$. To heuristically understand why this oscillation occurs, consider

the free system ($K = 1$) where, for a given a Fermi level μ , the Kramers' pair of states at the Fermi points are $e^{ik_F x}|\uparrow\rangle$, $e^{-ik_F x}|\downarrow\rangle$ [Fig. 1(a)]. In this basis, the superpositions $|\pm\rangle = \frac{1}{\sqrt{2L}}(e^{ik_F x}|\uparrow\rangle \pm e^{-ik_F x}|\downarrow\rangle)$ have magnetizations $m_{\pm} \sim \pm e^{-2ik_F x}$ and $M_z = 0$. A local magnetic field $H_x \delta(x)$ breaks the time-reversal symmetry of the pure QSH system, removes the degeneracy between the two states $|+\rangle$, $|-\rangle$ since $\langle +|H_x \delta(x)\sigma^x|+\rangle = -\langle -|H_x \delta(x)\sigma^x|-\rangle \neq 0$, and forces an incomplete compensation in magnetization. We emphasize that, in contrast to the oscillations that yield RKKY interactions, the helical nature of the QSH gives rise to spin oscillations in *direction* while the magnitude remains fixed.

In conclusion, we have shown that the nature of interacting helical edge states are unique in that they give rise to ordered oscillatory phases in the presence of finite chemical potential and spin imbalance. The experimental feasibility of realizing and detecting the SDW phase is promising. For instance, in HgTe quantum wells we expect the interactions to be weakly repulsive and $v \sim 10^5$ m/s which leads to a characteristic tunable wavelength of around $200/\mu$ nm where μ is the chemical potential tuned from the edge state Dirac point in meV. As mentioned, depending on the temperature this oscillation

will be modulated by a (perhaps strongly) decaying envelope function. To detect the oscillations, one could perhaps employ scanning tunneling microscopy as has been successfully used to observe the RKKY oscillation^{27,28} of the magnetization near a magnetic impurity, and Friedel oscillations²⁹ near a charged impurity.³⁰ While oscillations in the magnetization direction are harder to detect, any gate-voltage dependent oscillations would be indicative of our proposed SDW phase. The only modification to current setups would be the necessity of a back gate so that the oscillations could be accessed. Finally, the possibility to induce attractive interactions in the QSH system, as has been achieved in 1D cold atomic gases, would open up the fascinating prospect of realizing the chiral FFLO oscillatory superconducting phase.

We are grateful to W. DeGottardi for his involved discussions. This work was supported by the US Department of Energy, Division of Materials Sciences, under Award No. DE-FG02-07ER46453 (Q.L.M., T.L.H., S.V.), and by the NSF under Grant No. DMR 0758462 (T.L.H.), the AFOSR under Grant No. FA9550-10-1-0459 (M.J.G.), and the ONR under Grant No. N00141110728 (M.J.G.).

¹C. L. Kane and E. J. Mele, *Phys. Rev. Lett.* **95**, 226801 (2005).

²C. L. Kane and E. J. Mele, *Phys. Rev. Lett.* **95**, 146802 (2005).

³B. A. Bernevig and S. C. Zhang, *Phys. Rev. Lett.* **96**, 106802 (2006).

⁴B. A. Bernevig, T. L. Hughes, and S.-C. Zhang, *Science* **314**, 1757 (2006).

⁵M. König, S. Wiedmann, C. Brüne, A. Roth, H. Buhmann, L. Molenkamp, X.-L. Qi, and S.-C. Zhang, *Science* **318**, 766 (2007).

⁶C. Wu, B. A. Bernevig, and S.-C. Zhang, *Phys. Rev. Lett.* **96**, 106401 (2006).

⁷C. Xu and J. E. Moore, *Phys. Rev. B* **73**, 045322 (2006).

⁸X.-L. Qi, T. L. Hughes, and S.-C. Zhang, *Nat. Phys.* **4**, 273 (2008).

⁹L. Fu and C. L. Kane, *Phys. Rev. Lett.* **100**, 096407 (2008).

¹⁰V. Shivamoggi, G. Refael, and J. E. Moore, *Phys. Rev. B* **82**, 041405 (2010).

¹¹C.-X. Liu, T. L. Hughes, X.-L. Qi, K. Wang, and S.-C. Zhang, *Phys. Rev. Lett.* **100**, 236601 (2008).

¹²I. Knez, R. R. Du, and G. Sullivan, *Phys. Rev. Lett.* **107**, 136603 (2011).

¹³Y. Z. Ying Ran and A. Vishwanath, *Nat. Phys.* **5**, 298 (2010).

¹⁴P. Fulde and R. A. Ferrell, *Phys. Rev.* **135**, A550 (1964).

¹⁵A. I. Larkin and Yu. N. Ovchinnikov, *Sov. Phys. JETP* **20**, 762 (1965).

¹⁶K. Yang, *Phys. Rev. B* **63**, 140511 (2001).

¹⁷E. Zhao and W. V. Liu, *Phys. Rev. A* **78**, 063605 (2008).

¹⁸Y.-A. Liao, A. S. C. Rittner, T. Paprota, W. Li, G. B. Partridge, R. G. Hulet, S. K. Baur, and E. J. Mueller, *Nature (London)* **467**, 567 (2010).

¹⁹M. A. Ruderman and C. Kittel, *Phys. Rev.* **96**, 99 (1954).

²⁰T. Kasuya, *Prog. Theor. Phys.* **16**, 45 (1956).

²¹K. Yosida, *Phys. Rev.* **106**, 893 (1957).

²²C.-Y. Hou, E.-A. Kim, and C. Chamon, *Phys. Rev. Lett.* **102**, 076602 (2009).

²³T. Giamarchi, *Quantum Physics in One Dimension* (Oxford University Press, New York, 2003).

²⁴R. Egger and H. Grabert, *Phys. Rev. Lett.* **75**, 3505 (1995).

²⁵J. Maciejko, C. Liu, Y. Oreg, X.-L. Qi, C. Wu, and S.-C. Zhang, *Phys. Rev. Lett.* **102**, 256803 (2009).

²⁶Y. Tanaka, A. Furusaki, and K. A. Matveev, *Phys. Rev. Lett.* **106**, 236402 (2011).

²⁷F. Meier, L. Zhou, J. Wiebe, and R. Wiesendanger, *Science* **320**, 82 (2008).

²⁸L. Zhou, J. Wiebe, S. Lounis, E. Vedmedenko, F. Meier, S. Blügel, P. H. Dederichs, and R. Wiesendanger, *Nat. Phys.* **6**, 187 (2010).

²⁹J. Friedel, *Adv. Phys.* **3**, 446 (1954).

³⁰M. F. Crommie, C. P. Lutz, and D. M. Eigler, *Nature (London)* **363**, 524 (1993).