ECE 440
Lecture 39 : MOSFET-II

Class Outline:

• MOSFET - Qualitative
• Effective Mobility
• MOSFET - Quantitative
Key Questions

• How does a MOSFET work?
• Why does the channel mobility change and what causes it?
• What is the quantitative behavior of a MOSFET?
Now we wish to put our work on contacts and MOS capacitors together to understand how a MOSFET operates...

- We can modulate the current flowing from the source to the drain by using the gate and drain voltages.
- When the surface is inverted, a path exists for carriers to move to the drain.

To the left, we see a schematic of a MOSFET.
MOSFET - Qualitative

Let’s take a closer look at how the drain current should behave as a function of the terminal voltages...

Now begin to **increase the drain voltage**...

- Here the gate voltage is greater than the threshold voltage.
- The drain voltage is set to zero.
- A depletion region surrounds the device structure.

• Voltage drop associated with current flow negates inversion.
• Depletion widens and inversion carriers decreases.
• Channel conductance decreases
MOSFET - Qualitative

Keep increasing the drain voltage...

- Continued increases of the drain voltage causes continued reduction of the channel carriers.

- This causes a reduction in the channel conductance leading to a constant current.

- The greatest decrease happens near the drain where continued increases in the drain voltage will eventually eliminate the inversion layer.

- This is referred to as “pinch-off”.

- Increases in $V_d$ past pinch off move the pinched off region closer to the source.

- This region, devoid of carriers, absorbs most of the voltage drop.
MOSFET - Qualitative

So what do the I-V characteristics look like...

\[ V_T = 2\phi_F + \frac{K_S x_0}{K_O} \sqrt{\frac{4qN_A}{K_S \varepsilon_0}} \phi_F \quad \text{... ideal } n\text{-channel (p-bulk) devices} \]

\[ V_T = 2\phi_F - \frac{K_S x_0}{K_O} \sqrt{\frac{4qN_D}{K_S \varepsilon_0}(-\phi_F)} \quad \text{... ideal } p\text{-channel (n-bulk) devices} \]
Effective Mobility

But all of these models depend on the effective carrier mobility...

Remember the mobility?

We want to define a current, or charge per unit time crossing of observation orientated normal to the direction of current flow.

\[ J_{n}^{\text{drift}} = -qn\langle v_{dn} \rangle = qn\mu_{n}E \]

\[ J_{p}^{\text{drift}} = qp\langle v_{dp} \rangle = qp\mu_{p}E \]

Proportional to:
• Carrier drift velocity
• Carrier concentration
• Carrier charge
**Effective Mobility**

And from the definition of the current, we can define the mobility...

Using the definitions for the hole and electron drift currents:

\[
J_{n}^{\text{drift}} = -qn\langle v_{dn} \rangle = qn\mu_{n}E
\]

\[
J_{p}^{\text{drift}} = qp\langle v_{dp} \rangle = qp\mu_{p}E
\]

The electron and hole mobility then becomes:

Electron Mobility

\[
\mu_{n} = \frac{q\tau_{c}}{2m_{n}^{*}}
\]

Hole Mobility

\[
\mu_{p} = \frac{q\tau_{c}}{2m_{p}^{*}}
\]

What can we say about the mobility in general?

- Refers to the ease with which carriers move through a host crystal.

In units of:
Effective Mobility

What does temperature mean to mobility?

Silicon mobility at 300 K

Lattice Scattering
Ionized Impurity Scattering

Electrons
Holes

$\mu = \frac{10^{13}}{N_d + N_a}$

Electrical mobility decreases with increasing dopant concentration.
Effective Mobility

Seems like there are two main competing phenomena...

Impurity (dopant) Scattering:

\[ \mu_{\text{impurity}} \propto \frac{v_{th}^3}{N_A + N_D} \propto \frac{T^2}{N_A + N_D} \]

• The force acting on the particles is Coulombic.
• There is less change in the electron's direction of travel the faster it goes.

Phonon (lattice) Scattering:

\[ \mu_{\text{phonon}} = \frac{q \tau_{\text{phonon}}}{m^*} \propto \frac{1}{\#_{\text{phonons}} \times v_{th}} \propto \frac{1}{T \times T^{1/2}} \propto T^{-3/2} \]

Phonon scattering mobility decreases as the temperature increases.

Since the scattering probability is inversely proportional to the mean free time and the mobility, we can add individual scattering mechanisms inversely.

\[ \frac{1}{\mu} = \frac{1}{\mu_{\text{phonon}}} + \frac{1}{\mu_{\text{impurity}}} \]
Effective Mobility

We know **bulk mobility**, what makes the **channel mobility** different?

There are now **more scattering mechanisms** and the electrons are confined to a **smaller space** in the channel...

Source \( V_G \) Drain

We have additional **surface roughness** scattering...

SiO\(_2\) Si
**Effective Mobility**

What does the surface actually look like?

![Diagram showing electric field and surface](image)

- Electric field needed to invert surface pulls electrons closer to the gate interface.
- More field increases the interaction with the surface.
- This leads to a decrease in mobility not seen in the bulk.

<table>
<thead>
<tr>
<th>Parameters</th>
<th>Unit</th>
<th>$\Delta V_T$</th>
<th>$\Delta S_S$</th>
<th>$\Delta(I_{on}/I_{off})$</th>
</tr>
</thead>
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<td></td>
<td>(mV)</td>
<td>$%$</td>
<td>(mV/decade)</td>
<td>$%$</td>
</tr>
<tr>
<td>No SR</td>
<td>93.11</td>
<td>10.23</td>
<td>68.08</td>
<td>3.49</td>
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<tr>
<td>Fixed SR</td>
<td>127.5</td>
<td>14.9</td>
<td>70.33</td>
<td>3.45</td>
</tr>
</tbody>
</table>

Units: (mV), $\%$, (mV/decade), $\%$
Effective Mobility

We are also aware of the fact that there can be stray charges present in the insulating oxide...

- Alkali metals can easily be incorporated in the oxide during the fabrication process.
- These metals induce positive charges in the oxides which induce negative charges in the semiconductor.
- The magnitude of the effect depends on the number of sodium atoms and their distance from the surface.
- The atoms may drift in applied fields which leads to a continuous change in the threshold voltage.
Effective Mobility

What about oxide charges?

- Positive charges arise from interface states at the Si-SiO₂ interface.
- When oxidation is stopped, some ionic silicon is left near the surface.
- These ions along with other uncoupled bonds forms a sheet of positive charge at the interface.
- The charges depend on oxidation rate, heat treatment, and crystal orientation.

Why are devices made on [100] instead of [111]? Because interface charges are 10x higher on [111] relative to [100].
**Effective Mobility**

So how do they effect the **effective mobility**?

- **Coulomb interactions** between the electrons in the channel and the trapped oxide charges present another scattering rate not seen in the bulk.
- If we plot the effective carrier mobility in a MOSFET as a function of the average transverse electric field we get the **universal mobility degradation curve**.
- The curve is valid for any MOSFET independent of device and structural parameters such as oxide thickness.
Effective Mobility

So how do we determine the transverse electric field?

We determine the transverse field by applying Gauss' Law...

\[ Q = \int \int_D \cdot dS = \int \rho_v dv \]

From this we can find the average transverse field in the middle of the channel...

\[ \mathcal{E}_{eff} = \frac{1}{\varepsilon_s} \left( \frac{Q_d + \frac{1}{2} Q_n}{\varepsilon_s} \right) \]

We can often express the degradation by rewriting the drain current equation...

\[ I_D = \frac{\mu_n Z C_i}{L \{1 + \theta (V_G - V_T)\}} \left[ (V_G - V_T)V_D - \frac{1}{2} V_D^2 \right] \]

New term causes drain current to increase sub-linearly with gate bias for large biases.
Effective Mobility

But there is also a strong dependence of the longitudinal electric field...

• Carrier drift velocity increases linearly with electric field until it saturates.

• After saturation, mobility no longer makes any sense.

Let's describe the velocity in the following way...

\[ v = \mu \xi \text{ for } \xi < \xi_{sat} \]

and \( v = v_s \text{ for } \xi > \xi_{sat} \)

The maximum longitudinal field is the voltage drop near the drain end divided by the length of the pinch-off region.

\[ \xi_{max} = \left( \frac{V_D - V_D(\text{sat.})}{\Delta L} \right) \]

where

\[ \Delta L \approx \sqrt{2 t_{OX} x_j} \]
Before we begin to quantitatively express the behavior of the MOSFET, we need to realize something about the motion of the electrons...

Consider an n-channel device as shown to the right...

- $X$ is the depth measured from the oxide interface.
- $Y$ is the distance along the channel measured from the source.

Now average:

$$\bar{\mu}_n = \frac{\int_0^{x_c(y)} \mu_n(x, y)n(x, y) \, dx}{\int_0^{x_c(y)} n(x, y) \, dx}$$

Our in terms of the charge in the channel...

$$Q_N(y) = -q \int_0^{x_c(y)} n(x, y) \, dx$$
But this seems very difficult to use...

The mobility is a strong function of position within the channel. In our analysis, we assume that the effective mobility is independent of $V_D$ and channel position, $y$. This is true when the drain voltage is small. Here we have nearly constant charge in the channel.

However, we do know that there should be a dependence on the gate voltage...

$$\bar{\mu}_n = \frac{\mu_0}{1 + \theta(V_G - V_T)}$$

Here we use an empirical model to capture the dependence.
Now let’s begin analyzing the long-channel MOSFET quantitatively...

Let’s assume that:

1. We are above threshold
2. We are below pinch-off

Begin with the current density for electrons:

\[ J_N = q\mu_n n \mathcal{E} + qD_N \nabla n \]

Within the channel region, current flow is almost entirely along the y-direction and is mainly comprised of a drift component. Therefore, we neglect diffusion to obtain:

\[ J_N \approx J_{Ny} \approx q\mu_n n \mathcal{E}_y = -q\mu_n n \frac{d\phi}{dy} \]

Remember that each of these things is position dependent.
Since the vast majority of the current flows near the surface, we can now determine what the drain current ($I_D$) should be:

Let's find the current passing through a cross-sectional area...

$$I_D = - \int \int J_{Ny} \, dx \, dz = -Z \int_0^{x_c(y)} J_{Ny} \, dx$$

Plug in the position dependent terms...

$$I_D = \left( -Z \frac{d\phi}{dy} \right) \left( -q \int_0^{x_c(y)} \mu_n (x, y)n(x, y) \, dx \right)$$

$$I_D = - Z \mu_n Q_N \frac{d\phi}{dy}$$

But this is independent of $y$...

$$\int_0^L I_D \, dy = I_D L = - Z \int_0^{V_D} \bar{\mu}_n Q_N \, d\phi$$

$$I_D = - \frac{Z \bar{\mu}_n}{L} \int_0^{V_D} Q_N \, d\phi$$
However, we still need an expression which relates the charge to the channel potential...

To establish this relationship, remember what we know from MOS capacitors:

Above threshold, the charge on the gate is balanced in the semiconductor:

\[ \Delta Q_{\text{gate}} \left( \frac{\text{charge}}{\text{cm}^2} \right) = -\Delta Q_{\text{semi}} \left( \frac{\text{charge}}{\text{cm}^2} \right) \approx -Q_N \]

But with the charges being so close to the oxide, we can also say that:

\[ \Delta Q_{\text{gate}} \left( \frac{\text{charge}}{\text{cm}^2} \right) \approx C_o \Delta V_G = C_o (V_G - V_T) \]

Which can be used to find the final relationship between the channel potential and the charge in the channel...

\[ Q_N \approx -C_o (V_G - V_T) \]

with

\[ C_o \equiv \frac{C_O}{A_G} = \frac{K_O \epsilon_0}{x_0} \]
MOSFET - Quantitative

Let’s examine the applied gate voltage to begin to solidify our understanding...

• The negative workfunction difference causes the bands to be pulled down farther in equilibrium.

• To achieve flatband conditions, we must apply a positive voltage to overcome the inherent bending in the bands,

\[ V_G = V_{FB} - \frac{Q_s}{C_i} + \phi_s \]

The induced charge is composed of mobile charge, \( Q_n \), and fixed charge contributions, \( Q_d \).

We can substitute this relation into our equation and solve for the mobile charge...

\[ Q_n = -C_i \left[ V_G - \left( V_{FB} + \phi_s - \frac{Q_d}{C_i} \right) \right] \]
But we’re not quite to the point where we can easily write down the charge as a function of position...

We’re not dealing with this:

We have this instead:

So the situation is more like a resistive plate capacitor...

At the source: $\phi = V_G$

At the drain: $\phi = V_G - V_D$

So, let’s append our expression for the charge in the channel:

$$Q_N(y) \approx -C_0(V_G - V_T - \phi)$$
Now we can obtain the current–voltage relationship...

\[
Q_N(y) \equiv -C_o(V_G - V_T - \phi) \quad \rightarrow \quad I_D = -\frac{Z\mu_n C_o}{L} \int_0^{V_D} Q_N \, d\phi
\]

\[
I_D = \frac{Z\mu_n C_o}{L} \left[ (V_G - V_T)V_D - \frac{V_D^2}{2} \right] \left( \begin{array}{c} 0 \leq V_D \leq V_{Dsat} \\ V_G \geq V_T \end{array} \right)
\]

Be careful where you apply this... it's not for pinch-off or past saturation. Past saturation, however, we know the drain current doesn't increase. Therefore...

\[
I_{D|V_D \geq V_{Dsat}} = I_{D|V_D = V_{Dsat}} \equiv I_{Dsat} = \frac{Z\mu_n C_o}{L} \left[ (V_G - V_T)V_{Dsat} - \frac{V_{Dsat}^2}{2} \right]
\]

We can simplify this some because the channel charge tends to zero at \(V_{dsat}\)...

\[
Q_N(L) = -C_o(V_G - V_T - V_{Dsat}) = 0 \quad \rightarrow \quad V_{Dsat} = V_G - V_T
\]

\[
I_{Dsat} = \frac{Z\mu_n C_o}{2L} (V_G - V_T)^2
\]
MOSFET Quantitative

This makes sense based on what we already know about MOSFETs...

\[ I_D = \frac{\mu_n Z C_i}{L} [(V_G - V_T)V_D - \frac{1}{2}V_D^2] \]

For low drain voltages, the MOSFET looks like a resistor if the MOSFET is above threshold and depending on the value of \( V_G \).

Now we can obtain the conductance of the channel...

\[ g = \frac{\partial I_D}{\partial V_D} \approx \frac{Z}{L} \mu_n C_i (V_G - V_T) \]

• But again, this is only valid in the linear regime.
• We are assuming that \( V_D \ll V_G - V_T \).
So we can describe the **linear regime**, but how do we describe the **saturation regime**...

- As the drain voltage is increased, the voltage across the oxide decreases near the drain end.
- The resulting mobile charge also decreases in the channel near the drain end.
- To obtain an expression for the drain current in saturation, substitute in the **saturation condition**.

\[
I_D(\text{sat.}) \approx \frac{1}{2} \mu_n C_i \left( \frac{Z}{L} \right) (V_G - V_T)^2 = \frac{Z}{2L} \mu_n C_i V_D^2(\text{sat.})
\]

\[
g_m(\text{sat.}) = \frac{\partial I_D(\text{sat.})}{\partial V_G} \approx \frac{Z}{L} \mu_n C_i (V_G - V_T)
\]

\[
V_D(\text{sat.}) \approx V_G - V_T
\]