assume : i) LL - injection

ii) $W < x_b$

iii) no breakdown

No R-G → only diffusion

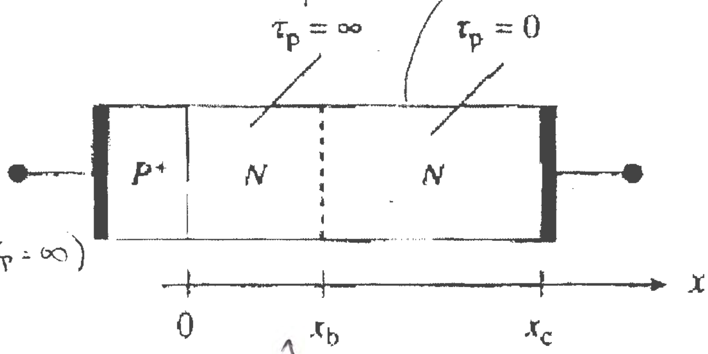
Problem #2:

$\Delta p_n = 0$

Steady state : $\dfrac{\partial \Delta p_n}{\partial t} = 0$

$G_L = 0$

$\dfrac{\Delta p_n}{\tau_p} \to 0 \quad (\tau_p = \infty)$



$\dfrac{\partial \Delta p_n}{\partial t} = 0$

$G_L = 0$

$\dfrac{\Delta p_n}{\tau_p} \Rightarrow 0$

only a few diffusion lengths away $(W \sim L_p)$

Consider the p$^+$-n junction pictured above and answer the following:

a. What does the I-V characteristic look like? Can you find an expression for it?

b. How does it change if we make $\tau_p > 0$ from $x_b < x < x_c$?

$$\frac{\partial^2 \Delta p_n}{\partial x^2} = 0 \qquad W \le x \le x_b$$

$$\boxed{\Delta p_n(x_b) = 0 \quad \& \quad \Delta p_n(W) = \frac{n_i^2}{N_D}\left(e^{\frac{qV_A}{kT}} - 1\right)}$$ Boundary Conditions

$$\boxed{\Delta p_n(x) = A_1 + A_2 x}$$

Apply BC

$$0 = A_1 + A_2 x \quad \text{and} \quad \Delta p_n(W) = A_1 + A_2 W$$

$$\Delta p_n(W) = -A_2(x_b - W) \quad \text{or} \quad \boxed{\begin{aligned} A_2 &= -\Delta p_n(W)/(x_b - W) \\ A_1 &= -A_2 x_b = \Delta p_n(W)\frac{x_b}{x_b - W} \end{aligned}}$$

$$\Delta p_n(x) = \Delta p_n(W)\left(\frac{x_b - x}{x_b - W}\right) = \frac{n_i^2}{N_D}\left(\frac{x_b - x}{x_b - W}\right)\left(e^{\frac{qV_A}{kT}} - 1\right)$$

$$J_p \approx -q D_p \frac{\partial \Delta p_n}{\partial x} = q\frac{n_i^2}{N_D}\left(\frac{\cancel{x_b}\, D_p}{x_b - W}\right)\left(e^{\frac{qV_A}{kT}} - 1\right)$$

(b) $\Delta p_n(x) = A_1 + A_2 x$ $\boxed{\Delta p_n(x_b) \neq 0}$ → new information

we still have: $\Delta p_n(w) = \frac{n_i^2}{N_D}\left(e^{qV_A/kT} - 1\right) = A_1 + A_2 W$

we can write: $\Delta p_n(x) = \underline{\Delta p_n(w)} + A_2(x - w)$ subs $x' = x - x_b$ ← coordinate shift, for $x' > 0$

$x_c - x_b \gg L_p$

$$\Delta p_n(x') = B e^{-x'/L_p}$$

Δp_n & J_p are continuous @ $x = x_b$ $J_p \propto \frac{\partial \Delta p_n(x)}{\partial x}$

$$\left.\begin{aligned} \Delta p_n(w) + A_2(x_b - w) &= B \\ A_2 &= -B/L_p \end{aligned}\right\} \text{continuity requirements}$$

$$\boxed{A_2 = \frac{\Delta p_n(w)}{[L_p + (x_b - w)]} \qquad B = \frac{\Delta p_n(w) L_p}{[L_p + (x_b - w)]}}$$

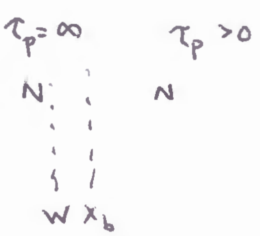
$$\Delta p_n(x) = \Delta p_n(w)\left[1 - \frac{x - w}{L_p + (x_b - w)}\right] \quad w \le x \le x_b$$

$$\Delta p_n(x') = \frac{\Delta p_n(w) L_p}{[L_p + (x_b - w)]} e^{-x'/L_p} \quad x' > 0$$

↓

$$J_p(x) \approx -q D_p \frac{\partial \Delta p_n}{\partial x} = q \frac{n_i^2}{N_D}\left(\frac{D_p}{L_p + (x_b - w)}\right)\left(e^{\frac{qV_A}{kT}} - 1\right)$$

$$J_p(x') \approx q \frac{n_i^2}{N_D}\left(\frac{D_p}{L_p + (x_b - w)}\right)\left(e^{\frac{qV_A}{kT}} - 1\right) e^{-x'/L_p}$$

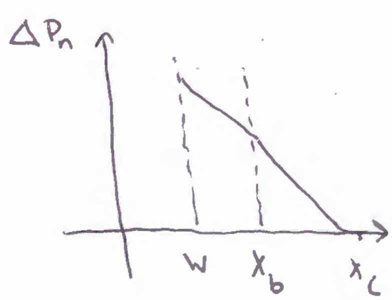


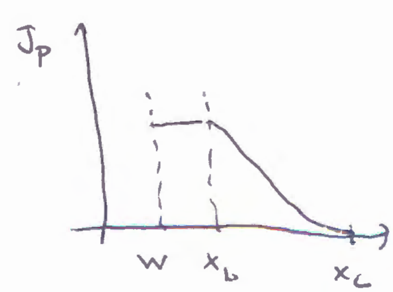
$$I = A J_p(x = w) = A J_p(x' = 0)$$

$$I = qA \frac{n_i^2}{N_D}\left(\frac{D_p}{L_p + (x_b - w)}\right)\left(e^{\frac{qV_A}{k_B T}} - 1\right)$$

$\Delta p_n(w) + A_2(x_b - w) = B e^{-(x - x_b)/L_p}$ (Δp_n)

$A_2 = -B/L_p$ $\left(\frac{\partial \Delta p_n(x)}{\partial x}\right)$





Problem #3:

The maximum power delivered by a solar cell can be found by maximizing the I-V product.

a. Show that maximizing the power leads to the expression: $\left(1+\frac{q}{kT}V_{mp}\right)e^{qV_{mp}/kT} = 1+\frac{I_{sc}}{I_{th}}$ where V_{mp} is the voltage for maximum power, I_{sc} is the magnitude of the short-circuit current, and I_{th} is the thermally induced reverse saturation current.

b. Assume a silicon solar cell with a dark saturation current I_{th} of 1.5 nA is illuminated such that the short-circuit current is I_{sc} = 100 mA. Use a graphical solution to obtain the voltage Vmp at the maximum power delivered.

(a)

$$I = I_{th}\left(e^{\frac{qV}{kT}}-1\right) - I_{op} = I_{th}e^{\frac{qV}{kT}} - I_{th} - I_{op}$$

$$\text{Power} = VI = VI_{th}e^{qV/kT} - VI_{th} - VI_{op}$$

$$\frac{\partial P}{\partial V} = \frac{qV}{kT}I_{th}e^{qV/kT} + I_{th}e^{qV/kT} - I_{th} - I_{op} = 0$$

$$= \left(\frac{qV}{kT}+1\right)I_{th}e^{qV/kT} = I_{th} + I_{op}$$

$$= \left(\frac{qV}{kT}+1\right)e^{qV/kT} = 1 + \frac{I_{op}}{I_{th}}$$

$$\boxed{I = 10^{-9}A \cdot e^{15.3} - 10^{-1}A = -96\,mA}$$

(b) $V_{mp} = 0.338$ V

rewrite as $\ln x = C - x$

$x = V_{mp}\cdot\frac{q}{k_BT}$

$$\ln\left(\frac{I_{sc}}{I_{th}}\right) = \ln\left(\frac{100\times10^{-3}A}{1.5\times10^{-9}A}\right) = 18$$

$$\ln\left(e^{\frac{qV_{mp}}{k_BT}}\cdot V_{mp}\cdot\frac{q}{k_BT}\right) = \ln\left(\frac{I_{sc}}{I_{th}}\right)$$

$$\ln\left(V_{mp}\cdot\frac{q}{k_BT}\right) + \frac{qV_{mp}}{k_BT} = \ln\left(\frac{I_{sc}}{I_{th}}\right)$$

$$\ln\left(V_{mp}\cdot\frac{q}{k_BT}\right) = \ln\left(\frac{I_{sc}}{I_{th}}\right) - \frac{qV_{mp}}{k_BT}$$

$$\ln(x) = 18 - x$$

$$V_{mp} = (15.3)(0.0255V)$$

$$\boxed{V_{mp} = 0.396V}$$