ECE 340
Lecture 19: Steady State Carrier Injection

Class Outline:

• Diffusion and Recombination
• Steady State Carrier Injection
Key Questions

- What are the major mechanisms of recombination?
- How do we account for recombination in our analysis?
- How does the diffusion current change when we keep the system perturbation?
- How far can a minority carrier diffuse?
Diffusion and Recombination

For the last 2 lectures we’ve been discussing diffusion...

Define the carrier flux for electrons and holes:

$$\phi_n(x) = -D_n \frac{dn(x)}{dx}$$

$$\phi_p(x) = -D_p \frac{dp(x)}{dx}$$

And the corresponding current densities associated with diffusion...

$$J_{diff}^n = qD_n \frac{dn(x)}{dx}$$

$$J_{diff}^p = -qD_n \frac{dp(x)}{dx}$$

Carriers move together, currents opposite directions.
Diffusion and Drift of Carriers

And what happens when drift and diffusion are occurring simultaneously...

The total current must be the sum of the electron and hole currents resulting from the drift and diffusion processes:

\[ J(x) = J_n + J_p \]

Where are the particles and currents flowing?

Dashed Arrows = Particle Flow  
Solid Arrows = Resulting Currents
Diffusion and Recombination

But from previous lectures, we know more in happening...

We have completely ignored recombination!!

Type 1: Direct recombination
- Electron and hole drift into the same vicinity and recombine.
- They can give off light if the semiconductor has a direct bandgap.

Type 2: R-G Center recombination
- R-G centers may be impurity atoms or lattice defects.
- Create states in the band gap.
- Electrons see a potential well and get trapped losing energy. Holes are attracted to the electron and annihilates it giving off heat to the lattice.

Type 3: Auger recombination
- Collision between two like carriers.
- Energy released by recombination is given to the surviving carrier.
- Surviving electron then loses excess energy through lattice collisions.
Diffusion and Recombination

So what does this mean?

Consider this semiconductor:

• The hole current density leaving the differential area may be larger or smaller than the current density that enters the area.
• This is a result of recombination and generation.

• Net increase in hole concentration per unit time, $dp/\text{dt}$, is the difference between hole flux per unit volume entering and leaving, minus the recombination rate.
Diffusion and Recombination

How can we explain this?

The net increase in hole concentration per unit time is the difference between the hole flux entering and leaving minus the recombination rate...

\[
\frac{\partial p}{\partial t} \bigg|_{x \to x + \Delta x} = \frac{1}{q} \left( J_P(x) - J_P(x + \Delta x) \right) \Delta x - \frac{\Delta p}{\tau_p}
\]

(1)

Rate of hole buildup. Increase in hole concentration in \( \Delta x A \) per unit time. Recombination rate

As \( \Delta x \) goes to zero, we can write the change in hole concentration as a derivative, just like in diffusion...

\[
\frac{\partial p(x,t)}{\partial t} = \frac{\partial p}{\partial t} = \frac{1}{q} \frac{\partial J_P}{\partial x} - \frac{\Delta p}{\tau_p}
\]

Holes

\[
\frac{\partial n(x,t)}{\partial t} = \frac{\partial n}{\partial t} = \frac{1}{q} \frac{\partial J_N}{\partial x} - \frac{\Delta n}{\tau_N}
\]

Electrons

These relations form the continuity equations.
**Diffusion and Recombination**

Are there any simplifications?

If the current is carried mainly by diffusion (small drift) we can replace the currents in the continuity equation...

\[
J_{\text{diff}}^n = qD_N \frac{\partial n}{\partial x} \\
J_{\text{diff}}^p = -qD_P \frac{\partial p}{\partial x}
\]

\[
J_n(x) = q\mu_n n(x)E(x) + qD_n \frac{dn(x)}{dx} \\
J_p(x) = q\mu_p p(x)E(x) - qD_p \frac{dp(x)}{dx}
\]

We put this back into the continuity equations...

\[
\frac{\partial p(x,t)}{\partial t} = \frac{\partial p}{\partial t} = -\frac{1}{q} \frac{\partial J_p}{\partial x} - \frac{\Delta p}{\tau_P}
\]

Diffusion equation for electrons

\[
\frac{\partial n(x,t)}{\partial t} = \frac{\partial n}{\partial t} = \frac{1}{q} \frac{\partial J_n}{\partial x} - \frac{\Delta n}{\tau_N}
\]

Diffusion equation for holes

Useful mathematical equation for many different physical situations...
Steady State Carrier Injection

To this point, we been assuming that the perturbation was removed...

What happens if we keep the perturbation?
• The time derivatives disappear

\[
\frac{\partial n}{\partial t} = D_N \frac{\partial^2 n}{\partial x^2} - \frac{\partial n}{\partial x} \tau_N
\]

\[
\frac{\partial p}{\partial t} = D_P \frac{\partial^2 p}{\partial x^2} - \frac{\partial p}{\partial x} \tau_P
\]

Where

Electrons
\[
\frac{d^2 n}{dx^2} = \frac{\Delta n}{D_N \tau_N} \equiv \Delta n \frac{1}{L_N^2}
\]

Holes
\[
\frac{d^2 p}{dx^2} = \frac{\Delta p}{D_P \tau_P} \equiv \Delta p \frac{1}{L_P^2}
\]

Where

\[
L_N = \sqrt{D_N \tau_N}
\]

\[
L_P = \sqrt{D_P \tau_P}
\]

Diffusion Length
Let’s consider the following situation...

Let’s assume that we are injecting excess holes into a sample of silicon.

• We do not remove the perturbation, so we maintain a constant excess hole concentration at the injection point, \( \delta p (x = 0) = \Delta p \).

• The injected holes then begin to diffuse along the bar recombining with a characteristic lifetime, \( \tau_p \).
Steady State Carrier Injection

What should we expect?

We expect the excess hole concentration to decay to zero at large distances from the perturbation.

\[ \delta p(x) = C_1 e^{L_p x} + C_2 e^{-L_p x} \]

Get constants from boundary conditions. Which are what?

The boundary conditions are:

\[ \delta p = 0 \text{ for large } x \]
\[ \delta p = \Delta p \text{ for } x = 0 \]

C1 = 0
C2 = \Delta p

Average distance a hole travels before it recombines.

\[ \delta p(x) = \Delta p e^{-x/L_p} \]
But we really want the average diffusion length...

What is the probability that an injected hole recombines in a particular interval?

We know that the probability that a hole injected at \( x = 0 \) survives to \( x \) is:

\[
\frac{p(x)}{\delta p} = e^{-\frac{x}{L_P}} \quad \text{Ratio of steady state concentrations}
\]

We know that the probability that a hole at \( x \) recombines in \( dx \) is:

\[
\frac{p(x) - p(x + \Delta x)}{p(x)} = -\frac{dp(x)}{dx} dx = \frac{dx}{L_P}
\]

Multiply the two probabilities...

\[
\left(e^{-\frac{x}{L_P}}\right) \left(\frac{dx}{L_P}\right) = \frac{1}{L_P} e^{-\frac{x}{L_P}} dx \quad \text{Compute expectation value}
\]

Distance minority carriers diffuse into a sea of majority carriers
The distribution of excess holes causes a current...

We have a diffusion current of holes moving from high concentration to low:

\[ J_{\text{diff}}^p = -qD_p \frac{\partial p}{\partial x} \]

Plug in what we know about the rate of change of the hole concentration with position:

\[ J_{\text{diff}}^p = -qD_p \frac{\partial \delta p}{\partial x} \]

Begin to reduce the equation...

\[ J_{\text{diff}}^p = q \frac{D_p}{L_p} \Delta p e^{-\frac{x}{L_p}} = q \frac{D_p}{L_p} \delta p(x) \]

The diffusion current depends on the excess carriers at a point and not on the initial concentration.